

# Length and Time Scales Relevant to Sound Generation in Excited Jets

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## Introduction

A GOOD deal of experimental and analytical research has been directed at the behavior of excited jets.<sup>1-4</sup> From an aeroacoustic point of view, these jets appear to point toward an avenue of noise reduction. Although Bechert and Pfizenmaier<sup>5</sup> reported a pronounced broadband lift of the radiated sound for excitation at the preferred Strouhal frequency ( $St = f_0 D / U_j \approx 0.5$ ), Moore<sup>6</sup> measured small reductions when the jet was driven at nondimensional frequencies well above 0.5.

The work of Kibens<sup>7</sup> suggests the possibility of significant reduction in the broadband noise; this is explored further herein. Measurements of certain length and time scales are presented. The results are applied to a simple analytical model of jet noise generation. Using explicit information about the spatial and temporal scales an effective source strength distribution is inferred, permitting (in principle) computation of the overall sound pressure level.

The measurements presented here suggest that only marginal reductions in broadband noise can be realized for jets operated at modest and high Reynolds numbers ( $Re > 300,000$ ).

## Theoretical Considerations

To a first approximation the generation of sound by turbulent flow is described by Lighthill's equation<sup>8</sup>:

$$\frac{1}{c_0^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\partial^2 \rho v_i v_j}{\partial x_i \partial x_j} \quad (1)$$

For unheated, subsonic jet flows the solution to Eq. (1) is expressed in the self and shear noise formalism due to Ribner<sup>9</sup>:

$$\begin{aligned} \bar{p}^2(x) = & \frac{\rho_0^2}{16\pi^2 c_0^4 |x|^2} \int \frac{\partial^4}{\partial \tau^4} \{ 4U_x(y + \frac{1}{2}\xi) U_x(y - \frac{1}{2}\xi) \\ & \times \overline{u_x u_x'}(y, \xi, \tau) + \overline{u_x^2 u_x''}(y, \xi, \tau) \} \delta[\tau - c_0^{-1}(\xi \cdot x)/|x|] \\ & \times d\tau d^3 \xi d^3 y \quad (u_x = u \cdot x/|x|) \end{aligned} \quad (2)$$

When the velocity correlations are described by a locally homogeneous, isotropic model with length and time scales  $L$  and  $T$  one can show that the far-field mean square sound pressure is

$$\bar{p}^2(x) = \frac{\rho_0^2}{c_0^4 |x|^2} \int_V \phi dV$$

$$\phi \propto \frac{\overline{u_j^2} U_j^2}{C^5} \frac{L^3}{T^4} \left\{ A \cos^2 \theta (1 + \cos^2 \theta) + \frac{\overline{u_j^2}}{U_j^2} B \right\} \quad (3)$$

with  $C = \{ [1 - 0.5(U_j/c_0) \cos \theta]^2 - 0.88(L/c_0 T)^2 \}^{1/2}$  and  $A$  and  $B$  constants of comparable magnitude. Thus the mean square pressure and the "effective source strength"  $\phi$  are

governed by the characteristic spatial and temporal scales. These may be inferred from measured two point space-time correlations.

Changes in these scales and the turbulence level  $\overline{u_j^2}$  will alter the acoustic intensity ( $\propto \bar{p}^2$ ). When maximizing broadband noise reduction one strives to diminish the turbulence level and the spatial coherence length  $L$ , and to augment the coherence time  $T$ . Whenever length and time scales behave in a similar manner, any potential benefit is reduced.

Physically the effective source volume, and hence the available potential energy, will diminish, if the length scales shrink. In addition enhanced temporal coherence makes the turbulent flow appear "more frozen": a frozen subsonically convected pattern does not radiate any sound at all.

Finally, a comment about the role of large-scale structures in the above analytical framework. The two-point correlations do not exclude any contributions from the large-scale structures; any periodic or pseudoperiodic patterns will impose their characteristic signatures on measured correlations. These will determine the asymptotic behavior of the space-time correlations for such structures may extend several jet diameters in the downstream direction.<sup>10</sup> Thus, a faithful model of the turbulence correlations will yield good estimates of the sound pressure.

## Discussion of Results

The relevant correlation measurements were performed on a 10 cm circular air jet operated at an exit velocity of 45 m/s, which corresponds to a Reynolds number of 300,000. The nozzle boundary layer thickness was 0.25 cm. The jet could be excited along the periphery of the nozzle lip by four 30 W speakers radiating into an annular cavity.

The flow was surveyed by two linearized DISA hot film probes and normalized correlations were computed by a PAR 101 correlator. A typical set of output data, along with a signal processing schematic, is shown in Fig. 1.

The envelope of the two point space-time correlations is a measure of the temporal coherence apparent to an observer moving at the local convection speed. The coherence time  $T$  is estimated by fitting the envelope with the function  $\exp(-a/|t|)$ , whence  $T = a^{-1}$ . The decaying exponential fits the data quite well. A similar procedure is applied to the longitudinal correlations. As only a measure of relative change is sought, these definitions are believed to suffice.

Three cases have been examined: the undisturbed jet, and excitation at Strouhal numbers 1.1 and 4.4. The former enhances the turbulence level, the latter brings about a decrease. (Resonances in the driver arrangement made  $St = 1.1$  preferable to  $St = 0.5$ ; the influence on the turbulence structure was similar.)

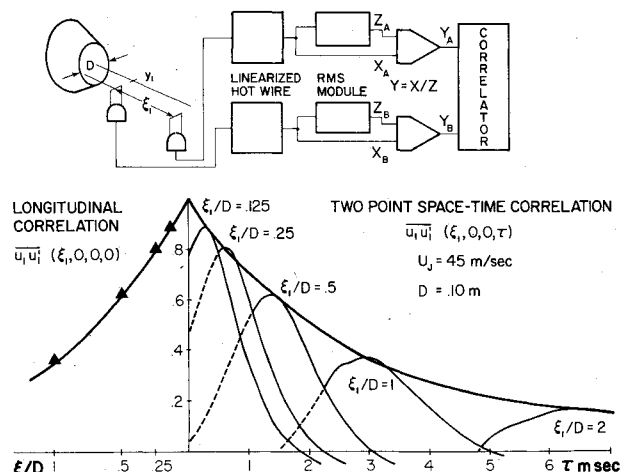


Fig. 1 Typical longitudinal and space-time velocity correlations measured with the experimental arrangement shown.

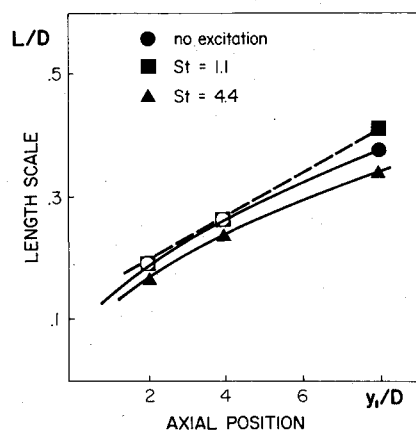


Fig. 2 Influence of acoustic excitation on axial length scale.

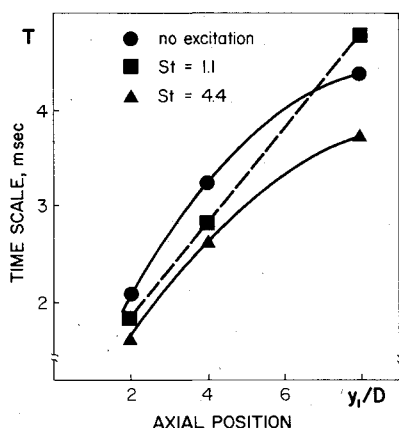


Fig. 3 Influence of acoustic excitation on the time scale.

At excitation levels of the order of 120 dB ( $\approx 1.6\%$  of  $0.5 \rho U^2$ ) the induced changes in the turbulence levels are approximately 10%. Greater percentage changes are realized near the potential core<sup>11</sup>; this region, however, does not contribute significantly to the radiated sound field.<sup>12</sup>

Figures 2 and 3 summarize the variations of length and time scales as functions of axial position and mode of excitation. The scales increase as the observer moves downstream. This is compatible with the notion that the jet flow diffuses and decelerates.

Excitation at  $St = 4.4$  reduces the turbulence levels as well as the length and time scales. Recall that a drop in turbulence level and length scale led to a decrease in the effective source strength; this beneficial effect must compete with the adverse influence of a diminished coherence time; at best, only marginal jet noise suppression can be expected. The present, albeit simplified, model predicts a 0.8 dB enhancement.

For the case of excitation at the moderate Strouhal number of 1.1, an increase in the turbulence level is observed. Length and time scales both exhibit similar behavior, and their influence on the radiated intensity is minimal. This results in an estimated augmentation of 3 dB in the overall sound pressure level. No acoustic measurements were taken to check the prediction, as the apparatus was located in a reverberant environment.

The present results suggest that the potential for turbulence control and the attendant reduction of jet noise is rather poor when a radial (pinching) mode of acoustic excitation is applied in the vicinity of the jet nozzle. Thus one should look to alternate methods of "tickling" the flow, so that the desired changes in the turbulence level, the length, and time scales are evoked.

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## Numerical Evaluation of Principal Value Integral by Gauss-Laguerre Quadrature

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### Introduction

**D**URING the past three decades, several numerical methods have been applied to the study of the hydrodynamics of a body floating on a free surface. These numerical methods fall into one of three groups, namely, multipole expansion, finite element (variational principle), and surface source distribution (Green's function).

Among the preceding methods, the surface source distribution method is favorable for a three-dimensional body of arbitrary shape in a uniform depth of water, however, a major difficulty has been encountered in applying this method. This difficulty resides in the evaluation of an improper integral containing a singularity in the Green's function.

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